Efficient Analysis of Microstrip Lines Including Edge Singularities in Spatial Domains

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***Abstract*—Microstrip lines are analyzed by considering edge-singular behavior using closed-form Green's functions in a spatial domain. A Maxwell function which incorporates the appropriate edge conditions of the line is introduced for the derivations of a transverse correlation function. From calculations of excess lengths of an open-end discontinuity, the results of the proposed method using the edge conditions are in better agreement with the quasistatic results than those of transverse uniform current variations for conductor strips with relatively wider width.**

***Index Terms*—Edge singularity, microstrip lines, transverse correlation function.**

I. Introduction

NUMERICAL analyzes of microstrip structures have many applications in designs of microwave integrated circuits. In particular, full-wave methods include the effects of surface wave, radiation loss, and coupling loss. In general, the most rigorous approaches characterizing the microstrip structures are to solve the system matrix based on either the electric field integral equation (EFIE) in the spectral domain or the mixed potential integral equation (MPIE) in the spatial domain. The full-wave methods involve time-consuming Sommerfeld integrals associated with highly oscillating and slowly decaying kernels. The MPIE provides relatively less singular kernels compared with the EFIE. In regions showing rapid variations in their current densities, however, a process of finely dividing such regions will improve the reliability of the numerical results of the MPIE. The increases in subsectional basis functions to model fine features of the current densities implies a large matrix dimension, leading to a longer computation time. Small cells along the edges of the microstrip transmission lines have been taken to consider the edge singularity of the line, with large width in the spatial domain, in order to add numerical accuracy, maintaining the number of basis function as little as possible [1].

In this letter, in order to consider edge singularities without using more basis functions along the transverse direction of microstrip lines in the spatial domain, a transverse correlation function is analytically derived using several mathematical properties of the Maxwell function with a singularity of order of one half and its Fourier conjugate. A convergence study based on the choices of the basis and testing functions in formulating the MoM matrix in [2] noted that basis functions with singularities of the order of less than one are admissible in the orthogonal direction of the current. A microstrip open-ended discontinuity will be investigated and compared with other results in order to validate the proposed method.

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II. Derivation of the Transverse Correlation Function

The formulations will be considered for microstrip lines with width w on a grounded dielectric substrate, for example, an open-end discontinuity shown in Fig. 1. A standard application of a Galerkins procedure for the MPIE in the spatial domain yields the general form of impedance matrix elements given by

$$Z_{mn} = \langle J_{xm}, G_{xx}^A * J_{xn} \rangle + \frac{1}{\omega^2} \left\langle J_{xm}, \frac{\partial}{\partial x}\left(G_q * \frac{\partial}{\partial x} J_{xn}\right)\right\rangle \tag{1}$$

where $\langle , \rangle$ implies the inner product, and J_n and J_m denote basis and testing functions, respectively. The spatial domain Green's functions G_{xx}^A and G_q can be determined with the help of the Sommerfeld identity after obtaining the coefficients and exponents by the two-level applications of the generalized pencil of functions (GPOF) method [3]. The basis function of (1), J_{xn}, is a rooftop function with a transverse dependence of $\sqrt{1-(2y/w)^2}$, which represents the real behavior of the current near the edge of the conducting strips.

$$\begin{aligned} J_{xn}(x,y) =& J_{xn}(x) \cdot J_t(y) \\ =& \begin{cases} \frac{\left(1-\frac{|x-x_n|}{\Delta x}\right)}{\sqrt{1-\left(\frac{2y}{w}\right)^2}}, & \text{if } \frac{|x-x_n|}{\Delta x} < 1 \text{ and } \left|\frac{2y}{w}\right| < 1 \\ 0, & \text{otherwise} \end{cases} \end{aligned} \tag{2}$$

where Δx is the half span of the rooftop function. The quadruple integrals in the impedance matrix elements are reduced to the double integrals of multiplication of Green's function and the correlation function after the changing of variables.

$$\langle J_{xm}, G_{xx}^A * J_{xn} \rangle = \iint du dv G_{xx}^A(u,v) CF(u,v) \tag{3}$$

with

$$CF(u,v) = \iint dx dy J_{xm}(x,y) J_{xn}(x-u, y-v)$$

where u and v mean the separation distance between the source and observation points for the x and y coordinates, respectively. The scalar potential part will be skipped here because it is similar to the vector part. The correlation function $CF(u,v)$ can be separated into longitudinal and transverse components for the structure assumed.

$$CF(u,v) = J_{xm} \otimes J_{xn} = CF_x(u) \cdot CF_y(v). \tag{4}$$

$CF_x(u)$ becomes a polynomial function with the order of 3 for the rooftop function. On the other hand, $CF_y(v)$ cannot be directly determined for the transverse variations considered. We try to find its analytic form using the property that the correlation of two functions in the spatial domain is equivalent to the multiplication of their conjugate pairs in the spectral domain.

$$\begin{aligned} CF_y(v) &= \int J_t^i(y) J_t^j(y-v)dy \\ &= \frac{\pi w^2}{4}\int_0^\infty J_0^2\left(\frac{w\alpha}{2}\right)\cos(\alpha v)d\alpha. \end{aligned} \tag{5}$$

The Fourier transform of $J_t(y)$ is $(w/2)J_0(w\alpha/2)$, where J_0 denotes a Bessel function of the zeroth-order. With the relation of [4, Eq. 6.672.6], (5) can be written as the Legendre function of the second kind

$$CF_y(v) = \frac{w}{2}Q_{-\frac{1}{2}}\left(1 - 2\left(\frac{v}{w}\right)^2\right), -w < v < w. \tag{6}$$

The Legendre function $Q_{-1/2}$ is given by the elliptic integral of the first kind with the relation of [5, Eq. 8.13.10], as shown below;

$$Q_{-\frac{1}{2}}(s) = K\left(\sqrt{\frac{1+s}{2}}\right), -1 < s < 1. \tag{7}$$

Now, the other form of $CF_y(v)$ is

$$CF_y(v) = \frac{w}{2}K\left(\sqrt{1 - \left(\frac{v}{w}\right)^2}\right), -w < v < w. \tag{8}$$

Instead of using the series form of the elliptic integral, it is more useful to take an approximation form of [4, Eq. 8.113.3] with a logarithmic function included. With K satisfactorily approximated as three terms shown in the appendix of [6], it becomes

$$\begin{aligned} CF_y(v) =& \frac{w}{2}\Bigg\{\left(1 + \frac{(v/w)^2}{4} + \frac{9}{64}\left(\frac{v}{w}\right)^4\right) \\ & \cdot \ln\frac{4w}{|v|} - \frac{(v/w)^2}{4} - \frac{21}{128}\left(\frac{v}{w}\right)^4\Bigg\}. \end{aligned} \tag{9}$$

It has an integrable logarithmic singularity at $v = 0$. It can be circumvented by means of a QDAGS routine in the IMSL package. The QDAGS is a general-purpose integrator appropriate for a function which may have endpoint singularities.

III. Numerical Results

In order to verify our derivations outlined in the previous section, we treated a microstrip open-end discontinuity in Fig. 1. The current of the strip of the open-end discontinuity can be expanded by a combination of semi-infinite microstrip current and rooftop subdomain current, which are described in detail in [7] and [8]. The transverse correlation function of (9) was applied in evaluating impedance elements Z_{mn} between rooftop basis functions. Matrix elements by the semi-infinite current can be found by a linear summation of impedance elements between rooftop functions Z_{mn} weighted by the sinusoidal values at sampling points of testing procedures. The open-end on a grounded dielectric slab we investigated has a width $w = 0.635$ mm, and a substrate thickness $d = 0.635$ mm with a relative permittivity $\epsilon_r = 9.9$. In Fig. 2, the magnitude and phase of the reflection coefficient by the proposed method are plotted with the quasistatic results using frequency-independent circuit models of [9], which was previously established to be in reasonably good agreement with experimental data and the results of other full-wave analyses [8]. A satisfactory agreement can be found within reasonable error bounds. Further, the open-end discontinuity is often modeled as having a reflection coefficient with a magnitude of less than unity due to radiation loss and a phase delay which can be explained by the length extension $\Delta l/d$.

$$\frac{1-\Gamma}{1+\Gamma} = g + j\tan(\beta\Delta l). \tag{10}$$

In Fig. 3, the excess length $\Delta l/d$ with respect to ratios of w/d at $f = 20$ GHz are plotted with the results of a usual uniform transverse cell for comparison. It is shown that the values of both cases give good agreement with quasistatic approximations of [9] for narrow widths with less than two substrate width and

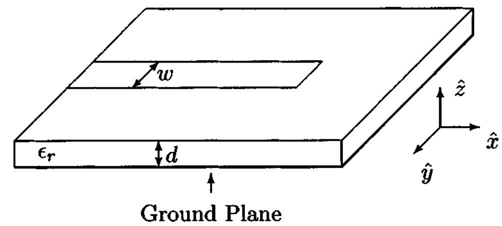

Fig. 1. Geometry of microstrip open-end discontinuity.

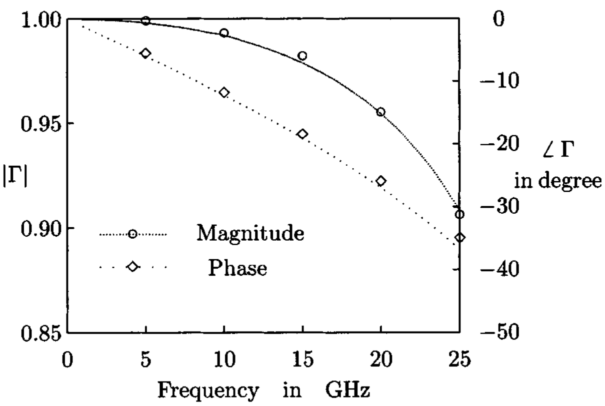

Fig. 2. Magnitude and phase of reflection coefficient for the open-end discontinuity (line: circuit model, marker: the proposed method).

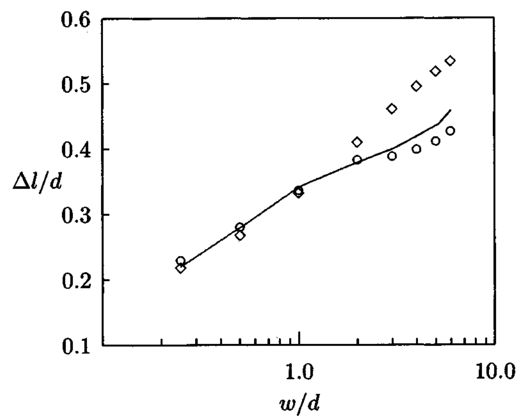

Fig. 3. Excess length versus the width of strip ($d = 0.635$ mm, $f = 20$ GHz) (line: circuit model, ○: edge cell, ◇: uniform cell).

then uniform case shift away as the ratio w/d becomes higher. Note that the edge cell gives improved results compared to the uniform cell for wider lines. At higher ratios, i.e., those with very small characteristic impedances, the discrepancy increases in both cases. The higher the ratio, the greater the discrepancy. Clearly, it can be known that the capacity of the edge cell to survive the increase in strip width is greater than that of the uniform cell. For improved results, it is important to choose the basis function after an examination of the strip widths. Also, the $\hat{y}$ current components should be additionally taken into account for wider strips.

IV. Conclusion

We developed a new, efficient, full-wave approach which considers the edge-singular behavior of microstrip lines in a spatial domain. A transverse correlation function was derived to include the edge behavior of microstrip lines with the help of tables of integrals. This was successfully verified through the analysis of a microstrip open-end discontinuity. From the calculations of the excess length of the microstrip open-end discontinuity, it was observed that the edge cell can survive the wider increase of the strip width, unlike the uniform cell. The method can be used in analyzes of strip structures such as strip resonators, gap discontinuities, microstrip feed lines, and strip antennas.

References

[1] R.-C. Hsieh and J.-T. Kuo, "Fast full-wave analysis of planar microstrip circuit elements in stratified media," *IEEE Trans. Microwave Theory Tech.*, vol. 46, pp. 1291–1297, Sept. 1998.

[2] M. I. Aksun and R. Mittra, "Choices of expansion and testing functions for the method of moments applied to a class of electromagnetic problems," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 503–509, Mar. 1993.

[3] M. I. Aksun, "Robust approach for the derivation of closed-form Green's functions," *IEEE Trans. Microwave Theory Tech.*, vol. 44, pp. 651–658, May 1996.

[4] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*. New York: Academic, 1980.

[5] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, National Bureau of Standards Applied Mathematical Series-55*. Washington, DC: U.S. Govt. Printing Office, 1966.

[6] R. E. Collin, *Foundations for Microwave Engineering*, 2nd ed, New York: McGraw-Hill, 1992.

[7] R. W. Jackson and D. M. Pozar, "Full-wave analysis of microstrip open-end and gap discontinuities," *IEEE Trans. Microw.ave Theory Tech.*, vol. MTT-33, pp. 1036–1042, Oct. 1985.

[8] H. Y. Yang, N. G. Alexopoulos, and D. R. Jackson, "Microstrip open-end and gap discontinuities in a substrate-superstrate structure," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 81–89, Oct. 1989.

[9] N. G. Alexopoulos and S. C. Wu, "Frequency-independent equivalent circuit model for microstrip open-end and gap discontinuities," *IEEE Trans. Microwave Theory Tech.*, vol. 42, pp. 1268–1272, July 1994.